AP Calculus BC

WS 74 - Sequences & Series

1) a) 3, 8, 13, 18, ... $a_n = -2 + 5n$

b) 5, −15, 45, −135 $a_n = (-1)^{n+1}(5)\cdot 3^{n-1}$

c) 1, 4, 9, 16, 25 $a_n = n^2$

2) a) $\sum_{n=1}^{\infty} \frac{1+3n^2+n^3}{4n^3-5n+2} = S$

$\lim_{n\to\infty} \frac{1+3n^2+n^3}{4n^3-5n+2} = \frac{1}{4}$

S diverges by n^{th} term test

b) $\sum_{n=1}^{\infty} \frac{1}{n^2} = S$

$\lim_{n\to\infty} \frac{1}{n^2} = 0$

S possibly converges

c) $\sum_{n=1}^{\infty} \frac{n!}{2n!+1} = S$

$\lim_{n\to\infty} \frac{n!}{2n!+1} = \frac{1}{2}$

S diverges by n^{th} term test

d) $\sum_{n=1}^{\infty} \frac{(n+2)!}{10n!} = S$

$\lim_{n\to\infty} \frac{(n+2)(n+1)\cdot n\cdot(n-1)\cdots 3\cdot 2\cdot 1}{10\,(n\cdot(n-1)\cdots 3\cdot 2\cdot 1)}$

$\lim_{n\to\infty} \frac{(n+2)(n+1)}{10} = \infty$

S diverges by n^{th} term test

3) (a) $3 + \frac{15}{4} + \frac{17}{16} + \frac{375}{64} + \cdots = S$

$r = \frac{\frac{15}{4}}{3} = \frac{15}{12} = \frac{5}{4}$

Since $|r| > 1$, S diverges by GST

(b) $S = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \cdots$

$r = \frac{1/4}{1/2} = 1/2$

Since $|r| < 1$, S converges by GST

$S = \frac{a_1}{1-r} = \frac{1/2}{1-1/2} = 1$

4) a) $\sum_{n=1}^{\infty} \frac{2n+3}{3n-5} = S$

$\lim_{n\to\infty} \frac{2n+3}{3n-5} = \frac{2}{3}$

S diverges by n^{th} term test

b) $\sum_{n=1}^{\infty} \frac{n!}{2n!+1} = S$

$\lim_{n\to\infty} \frac{n!}{2n!+1} = \frac{1}{2}$

S diverges by n^{th} term test

c) $\sum_{n=1}^{\infty} \frac{3^n-2}{3^n} = S$

$\lim_{n\to\infty} \frac{3^n-2}{3^n} = 1$

S diverges by n^{th} term test

d) $\sum_{n=2}^{\infty} \frac{1}{(1.1)^n} = \sum_{n=2}^{\infty} \frac{1}{(11/10)^n} = \sum_{n=2}^{\infty} \left(\frac{10}{11}\right)^n = S$

Since $|r|<1$, S converges by GST

$S = \frac{100/121}{1 - 10/11} = \frac{100/121}{1/11} = \frac{100}{11}$